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Expected value and variance of insurance surplus for a portfolio of equity-linked policies

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Summary: In this paper, we analyze the first two moments of insurance surplus for a portfolio of Equity linked policies. We adopt a definition of surplus as the difference between the retrospective gain and prospective loss: if we fix a valuation date r, the accumulated value to time r of the insurance cash flows that occurred between times 0 and r represents the retrospective gain and the present value at time r of the cash flows that occur after r is the prospective loss. Numerical application illustrates the results.

Keywords: Equity-linked, Prospective Loss, Retrospective Gain, Surplus.

1. Introduction

This paper proposes a model to compute the expected value and variance of insurance surplus for a portfolio of Equity-linked policies. In the actuarial literature, there are many contributions on pricing models for valuating this kind of product, considering both single and periodic premiums. Boyle and Schwartz (1997) extend the Black-Scholes (1973) framework to insurance contracts and provide theoretical basis for pricing death benefit in equity-linked contract. The main difference is that in the case of insurance products linked to financial market the fee is deducted on an ongoing basis as a proportion of the value of the underlying assets, while in the Black and Scholes approach the option premium is paid up-front. Delbaen (1990) and Aase and Persson (1994) price equity-linked contracts with periodic premiums under the assumption of deterministic interest rates; Bacinello and Ortu (1994) analyze the case of stochastic interest rates and derive a closed pricing formula for policies with single premiums. Bacinello (2004) consider surrender option in equity-linked contract; Vannucci (2003a, 2003b) analyze the presence of minimal return guarantees in unit linked policies.

With respect to the cited papers, we assume a different point of view and fix our attention on Surplus valuation. In this regard, we look at the actuarial research literature on insurance surplus and insolvency probability (Coppola et al. (2003), Dahl (2004), Hoedemakers et al. (2005), Lysenko and Parker (2007), Marceau and Gaillardetz (1999) and Parker (1994; 1996). The abovementioned papers deal with the stochastically discounted value of future cash flows in respect of life insurance and life annuity contracts. We apply this methodology to Unit Linked policies, extending the models appearing in the literature in order to study a product with a payments linked to a fund account. In the manner of Lysenko and Parker (2007), we adopt a definition of surplus as the difference between the retrospective gain and prospective loss: if we fix a valuation date r, the accumulated value to time r of the insurance cash flows that occurred between times 0 and r represents the retrospective gain and the present value at time r of the cash flows that occur after r is the prospective loss. We modify the model proposed by Lysenko and Parker (2007) in order to capture the uncertainty of a death benefit linked to a fund account.

The paper is organized as follows: in section 2 we describe the model; in section 3 we define the surplus as the difference between the retrospective gain and prospective loss and derive the first two moments of its distribution. Financial hypotheses are described in section 4. Numerical results are shown in section 5. Concluding remarks are discussed in section 6.

2. The model

In this work, we consider a portfolio of identical equity-linked policies, which are issued to a group of m policyholders who are aged x

with the same risk characteristics, and whose survival probability distribution are independent and identical; the final age is *n*.

Unit linked contracts can be structured in different ways: both of the constituent living and death benefits or just one of them can be linked to a fund account. We analyze the latter case and consider only the death benefit invested in a fund, while the living benefit represented by a conventional life annuity with annual payment *R*. Consequently, the premium can be ideally decomposed into a sterling part and a unit part:

$$P = P^{'} + P^{''} \tag{1}$$

where P' is the sterling part, relating to the annuity, and P'' is the unit part, which is invested in a fund. We consider a single premium paid at time 0.

Let V_t be the value of the account at time t, which is linked to a unit fund. Following the standard assumptions in the literature, we model the evolution of the account value as:

$$dV_t = (\mu - \eta) V_t dt + \sigma V_t dW_t \tag{2}$$

where W_t is a standard Brownian motion under the real probability space, μ is the drift rate, η is the insurance fee. The risk neutral process for V_t is:

$$dV_t = (r - \eta) V_t dt + \sigma V_t dW_t^Q \tag{3}$$

where r is the risk free rate and W_t^Q is a Brownian motion under a new Girsanov transformed measure Q.

The premium is calculated according to the equivalence principle:

$$P = Ra_{n,i} + D_0 \tag{4}$$

where $a_{n,i}$ is the actuarial value of an annuity, *i* is the technical rate used to price the annuity and D_0 is the value of death benefit $t = 0$, with

$$D_0 = E_t \{ E_0 \{ V_\tau | \tau = t \} \} \tag{5}$$

where τ is the random time of death.

Let r be a valuation date at which we estimate the surplus linked to this contract.

Let $RC_j^{(r)}$ be the net cash flow at time j for $0 \le j \le r$; it is called retrospective cash inflow at time r. It is given by:

$$\begin{aligned} RC_j^{(r)} &= \sum_{i=1}^{m} \left[P \cdot 1_{\{j=0\}} - R \cdot \alpha_{i,j} 1_{\{j\rangle 0\}} - V_j \cdot \delta_{i,j} 1_{\{j\rangle 0\}} \right] = \\ &= m \cdot P 1_{\{j=0\}} - R \left(\sum_{i=1}^{m} \alpha_{i,j} \right) 1_{\{j\rangle 0\}} - V_j \left(\sum_{i=1}^{m} \delta_{i,j} \right) 1_{\{j\rangle 0\}} = \\ &= m \cdot P 1_{\{j=0\}} - R \cdot \alpha_j 1_{\{j\rangle 0\}} - V_j \cdot \delta_j 1_{\{j\rangle 0\}} \end{aligned} \tag{6}$$

where $1_A : X \to \{0,1\}$ is the characteristic function defined on X as:

$$1_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A \end{cases};$$

$$\alpha_{i,j} = \begin{cases} 1 \text{ if policyholder i is alive at time j ;} \\ \quad 0 \text{ otherwise} \end{cases}$$

$$\delta_{i,j} = \begin{cases} 1 \text{ if policyholder i is dies in year j} \\ \quad 0 \text{ otherwise} \end{cases}$$

α_j is the number of people from the initial group of m policyholder who survive to time j and δ_j is the number of deaths in year j. Let m_r be the size of the portfolio at time r; for $0 < j \le n - r$ the number of survivors(deaths) at time j given the number of survivors(deaths) at time m follows a binomial distribution:

$$\{\alpha_j | \alpha_r = m_r\} \approx BIN(m_r, {}_j p_{x+r})$$
$$\{\delta_j | \alpha_r = m_r\} \approx BIN(m_r, {}_{j-1/1} q_{x+r})$$

where ${}_j p_{x+r}$ is the probability for a policyholder aged $x+r$ to be alive at time j and ${}_{j-1/1}q_{x+r}$ is the probability for a policyholder aged $x+r$ to die between $j-1$ and j.

We consider $r=0$, since we study all cash flows as viewed from time 0. We have for $k<j$:

$$E_0\left[\alpha_{i,j}\right]=m\cdot {}_j p_{x\,i}; \qquad E_0\left[\delta_{i,j}\right]=m\cdot {}_{j-1/1}q_{x_i}$$

$$Var_0\left[\alpha_{i,j}\right]=m\cdot {}_j p_{x\,i}(1-{}_j p_{x\,i}); \qquad Var_0\left[\delta_{i,j}\right]=m\cdot {}_{j-1/1}q_{x_i}(1-{}_{j-1/1}q_{x_i})$$

$$Cov_0\left[\alpha_{i,k},\alpha_{i,j}\right]=m\cdot {}_j p_{x\,i}(1-{}_k p_{x\,i})$$

$$Cov_0\left[\delta_{i,k},\delta_{i,j}\right]=-m\cdot {}_{j-1/1}q_{x_i}\cdot {}_{j-1/1}q_{x_i}$$

$$Cov_0\left[\delta_{i,j},\alpha_{i,j}\right]=-m\cdot {}_{j-1/1}q_{x_i}\cdot {}_j p_{x\,i}$$

$$Cov_0\left[\delta_{i,k},\alpha_{i,j}\right]=-m\cdot {}_{j-1/1}q_{x_i}\cdot {}_j p_{x\,i}$$

$$Cov_0\left[\alpha_{i,k},\delta_{i,j}\right]=m\cdot(1-{}_k p_{x\,i})\,{}_{j-1/1}q_{x_i}$$

Calculation of the cash flow moments is straightforward. Under the reasonable assumption of independence between V_j and δ_j and between V_j and α_j we have:

$$E\left[RC_j^{(r)}\right]=m\cdot P1_{\{j=0\}}-R\cdot E\left[\alpha_j\right]1_{\{j\rangle 0\}}-E\left[V_j\right]E\left[\delta_j\right]1_{\{j\rangle 0\}} \tag{7}$$

Moreover, we can calculate the variance of the retrospective cash flow:

$$Var\left[RC_j^r\right]=R^2Var\left[\alpha_j\right]1_{\{j\rangle 0\}}+Var\left[V_j\delta_j\right]1_{\{j\rangle 0\}}+2RCov\left[\alpha_j,V_j\delta_j\right]1_{\{j\rangle 0\}} \tag{8}$$

and the covariance of the retrospective cash flows:

$$Cov\left[RC_k^{(r)},RC_j^{(r)}\right]=$$
$$=R^2Cov\left[\alpha_k,\alpha_j\right]+Cov\left[V_k\delta_k,V_j\delta_j\right]+RCov\left[\alpha_k,V_j\delta_j\right]+RCov\left[\alpha_j,V_k\delta_k\right] \tag{9}$$

Now we fix our attention on the time period after *r*. Let $PC_j^{(r)}$ be the net cash flow plus the value of the shares invested in the fund that occurs *j* time units after *r* for $0 \le j \le n-r$, where *n* is the final age underlying the life table; this is called the prospective cash outflow at time *r*. It is given by:

$$\begin{aligned} PC_j^{(r)} &= \sum_{i=1}^{m} \left[R \cdot \alpha_{i,(r+j)} 1_{\{j \rangle 0\}} + V_{r+j} \cdot \delta_{i,(r+j)} 1_{\{j \rangle 0\}} \right] = \\ &= R \left(\sum_{i=1}^{m} \alpha_{i,(r+j)} \right) 1_{\{j \rangle 0\}} + V_{r+j} \left(\sum_{i=1}^{m} \delta_{i,(r+j)} \right) 1_{\{j \rangle 0\}} = \\ &= R \cdot \alpha_{r+j} 1_{\{j \rangle 0\}} + V_{r+j} \cdot \delta_{r+j} 1_{\{j \rangle 0\}} \end{aligned} \tag{10}$$

We can derive formulae for the moments of the cash flow in the same manner as before. Specifically:

$$E\left[PC_j^{(r)}\right] = R \cdot E\left[\alpha_{r+j}\right] 1_{\{j \rangle 0\}} + E\left[V_{r+j}\right] E\left[\delta_{r+j}\right] 1_{\{j \rangle 0\}} \tag{11}$$

$$Var\left[PC_j^r\right] = R^2 Var\left[\alpha_{r+j}\right] 1_{\{j \rangle 0\}} + Var\left[V_{r+j}\delta_{r+j}\right] 1_{\{j \rangle 0\}} + 2R Cov\left[\alpha_{r+j}, V_{r+j}\delta_{r+j}\right] 1_{\{j \rangle 0\}} \tag{12}$$

$$\begin{aligned} Cov\left[PC_j^r, PC_k^r\right] &= R^2 Cov\left[\alpha_{r+k}, \alpha_{r+j}\right] + Cov\left[V_{r+k}\delta_{r+k}, V_{r+j}\delta_{r+j}\right] + \\ &+ R \cdot Cov\left[\alpha_{r+k}, V_{r+j}\delta_{r+j}\right] + R \cdot Cov\left[\alpha_{r+j}, V_{r+k}\delta_{r+k}\right] \end{aligned} \tag{13}$$

Next, we introduce two random variables, the retrospective gain and the prospective loss, which will be used to define the surplus.

3. Retrospective gain, prospective loss and surplus

The *Retrospective Gain* at time r is the difference between the accumulated value to time r of past premiums collected and benefits paid. It can be expressed in terms of $RC_j^{(r)}$ as follows:

$$RG_r = \sum_{j=0}^{r} RC_j^r e^{I(j,r)} \tag{14}$$

where *I(s,r)* denotes the force of interest accumulation function between times *s* and *r* if $0 \le s \le r$ and the force of interest actualization function if $r \le s \le n-r$; it is given by:

$$I(s,r) = \begin{cases} \sum_{j=s+1}^{r} \lambda(j) & \text{if } s < r \\ 0 & \text{if } s = r \\ -\sum_{j=s}^{r+1} \lambda(j) & \text{if } s > r \end{cases}$$

and $\lambda(j)$ is the force of interest in period $(j-1, j]$.

We assume that future lifetimes and rates of return are independent; and, in addition, for the sake of simplicity, we assume independence between the fund value and interest rate. Thus, we obtain:

$$E[RG_r] = \sum_{j=0}^{r} E[RC_j^r] E[e^{I(j,r)}] \tag{15}$$

$$\begin{aligned} Var[RG_r] &= E[RG_r^2] - [E[RG_r]]^2 = \\ &= \sum_{k=0}^{r}\sum_{j=0}^{r} E[RC_k^r RC_j^r] E[e^{I(k,r)+I(j,r)}] - \left\{\sum_{j=0}^{r} E[RC_j^r] E[e^{I(j,r)}]\right\}^2 = \\ &= \sum_{k=0}^{r}\sum_{j=0}^{r} \{Cov[RC_k^r, RC_j^r] + E[RC_k^r] E[RC_j^r]\} E[e^{I(k,r)+I(j,r)}] + \\ &- \left\{\sum_{j=0}^{r} E[RC_j^r] E[e^{I(j,r)}]\right\}^2 \end{aligned} \tag{16}$$

The *Prospective Loss* at time *r* is the difference between the discounted values to time r of future benefits to be paid and premiums to be collected (although, in this case, there are no future premiums since the contract has a single premium at time 0). The Prospective Loss can be expressed in terms of $PL_j^{(r)}$ as follows:

$$PL_r = \sum_{j=0}^{n-r} PC_j^r e^{I(r,r+j)} \tag{17}$$

The moments of PL_r can be calculated in a similar way to the moments of RG_r.

At this point of the analysis, we define the net stochastic *Surplus* as the difference between the retrospective gain and the prospective loss:

$$S_r = RG_r - PL_r = \sum_{j=0}^{n} FC_j^r e^{I(j,r)} \tag{18}$$

where FC_j^r is the generic cash flow (outflow or inflow) at time j.

Thanks to our previous results, we can calculate the expected value and variance of surplus per policy:

$$E[S_r / m] = E[RG_r / m] - E[PL_r / m] = \frac{1}{m}\sum_{j=0}^{n} E[FC_j^r]E[e^{I(j,r)}] \tag{19}$$

$$Var[S_r / m] = Var\left[\sum_{j=0}^{n} \frac{FC_j^r e^{I(j,r)}}{m}\right] =$$

$$= \frac{1}{m^2}\left\{\sum_{j=0}^{n} Var[FC_j^r e^{I(j,r)}] + \sum_{j=0}^{n}\sum_{\substack{k=0 \\ k\neq j}}^{n} Cov\left(FC_j^r e^{I(j,r)}, FC_k^r e^{I(k,r)}\right)\right\} \tag{20}$$

In the following we develop the previous formulas.

The variance of the cash flows (both retrospective or prospective) is given by the following formula:

$$Var[FC_j^r] = R^2 Var[\alpha_j] + Var[V_j \delta_j] + 2R \cdot Cov[\alpha_j, V_j \delta_j]$$

where:

$$Var[V_j \delta_j] = E[V_j^2]E[\delta_j^2] - (E[V_j])^2 (E[\delta_j])^2 ;$$

$$Cov[\alpha_j, V_j \delta_j] = E[V_j]E[\alpha_j \delta_j] - E[\alpha_j]E[V_j]E[\delta_j] = E[V_j]Cov[\alpha_j, \delta_j]$$

The covariance of the cash flows is:

$$Cov[FC_k^{(r)}, FC_j^{(r)}] =$$

$$= R^2 Cov[\alpha_k, \alpha_j] + Cov[V_k \delta_k, V_j \delta_j] + RCov[\alpha_k, V_j \delta_j] + RCov[\alpha_j, V_k \delta_k]$$

Where:

$$Cov[V_k\delta_k, V_j\delta_j] = E[V_k\delta_k V_j\delta_j] - E[V_k\delta_k]E[V_j\delta_j] =$$
$$= E[V_k]E[V_j]E[\delta_k\delta_j] - E[V_k]E[\delta_k]E[V_j]E[\delta_j] = E[V_k]E[V_j]Cov[\delta_k, \delta_j]$$

$$Cov[\alpha_k, V_j\delta_j] = E[V_j]Cov[\alpha_k, \delta_j]$$

$$Cov[\alpha_j, V_k\delta_k] = E[V_k]Cov[\alpha_j, \delta_k]$$

Finally, the variance of the surplus can be calculated. Under the assumption that the values of the force of the interest at different time points are independent and identical distributed the variance of the surplus is the following:

$$Var[S_r / m] = Var\left[\sum_{j=0}^{n} \frac{FC_j^r e^{I(j,r)}}{m}\right] =$$
$$= \frac{1}{m^2}\left\{\sum_{j=0}^{n} Var\left[FC_j^r e^{I(j,r)}\right] + \sum_{j=0}^{n}\sum_{\substack{k=0 \\ k\neq j}}^{n} Cov\left(FC_j^r e^{I(j,r)}, FC_k^r e^{I(k,r)}\right)\right\}$$

where:

$$Var\left[FC_j^r e^{I(j,r)}\right] =$$

$$= E\left[FC_j^2 e^{I(j,r)2}\right] - \left\{E\left[FC_j^r e^{I(j,r)}\right]\right\}^2 = E\left[FC_j^2\right]E\left[e^{I(j,r)2}\right] - E\left[FC_j\right]^2 E\left[e^{I(j,r)}\right]^2$$

$$Cov\left[FC_j^r e^{I(j,r)}, FC_k^r e^{I(k,r)}\right] =$$

$$= E\left[FC_j^r FC_k^r e^{I(j,r)+I(k,r)}\right] - E\left[FC_j^r\right]E\left[e^{I(j,r)}\right]E\left[FC_k^r\right]E\left[e^{I(k,r)}\right] =$$
$$= E\left[e^{I(j,r)}\right]E\left[e^{I(k,r)}\right]Cov\left[FC_j^r, FC_k^r\right]$$

4. Financial hypotheses

In accordance with the Black & Scholes' framework, we model the evolution of the unit fund as in (1). Since W_t is a standard Brownian motion, it follows that:

$$E_0[V_j] = V_0 \exp\{(\mu - \eta) j\}$$

$$E_0[V_j^{\,2}] = V_0^2 \exp\{2(\mu - \eta) j + \sigma^2 j\}$$

In order to be consistent with this framework, we hypothesize that λ(j) for each j are independent and identical normally distributed random variables: $\lambda(j) \sim N(\beta,\varepsilon^2)$; and this implies that $I(r,s)$ is a normal random variable: $I(j,r) \sim N((r-j-1)\beta,(r-j-1)\varepsilon^2)$ and that both discount and accumulation functions follow a lognormal distribution.

Since $Y \sim N(E[Y],Var[Y])$, then the m^th^–moment of e^Y is:

$$E[e^{mY}] = e^{mE[Y] + \frac{m^2}{2} Var[Y]}$$

and we can easily find the moments of $e^{I(j,r)}$:

$$E[e^{I(j,r)}] = e^{(r-j-1)\beta + \frac{r-j-1}{2}\varepsilon^2}$$

$$Var[e^{I(j,r)}] = e^{2(r-j-1)\beta + 2(r-j-1)\varepsilon^2} - e^{2(r-j-1)\beta + (r-j-1)\varepsilon^2}$$

$$Cov[e^{I(k,r)}, e^{I(j,r)}] = 0$$

$$E[e^{I(k,r)+I(j,r)}] = E[e^{I(k,r)} e^{I(j,r)}] = Cov[e^{I(k,r)}, e^{I(j,r)}] + E[e^{I(k,r)}]E[e^{I(j,r)}] =$$
$$= E[e^{I(k,r)}]E[e^{I(j,r)}]$$

5. Numerical results

The mathematical efforts of the previous section are directed to analyze the evolution of the surplus in terms of expected value and possible fluctuations at each date. In this section, we apply the model and show numerical results for a portfolio of identical Variable Annuities with a GMDB option. We consider a group of 10000 policyholders aged 50 with the same risk characteristics, whose survival probability distributions are independent and identically. The mortality table used in our calculation is the SIM2002 based on the Italian male population, with the maximum age fixed as 110.

The product comprises a deferred annuity, with annual payment equal to 1, and a GMDB option. The single premium is paid at time 0; and it can be decomposed into two parts: P' is the sterling part, relating to the annuity, and P'' is the unit part, relating to the option. In particular, according to the equivalence principle $P' = a_{(110-50),i}$; in addition, we consider $P''=1$ the amount invested into the fund. We set: $i=4.5\%$, $\mu\text{-}\delta = 5\%$, $\sigma^2=3\%$, $\beta=4\%$ and $\varepsilon^2=1.5\%$.

We evaluate the first 2 moments of the surplus at different dates r and show the results in Figures 1 and 2.

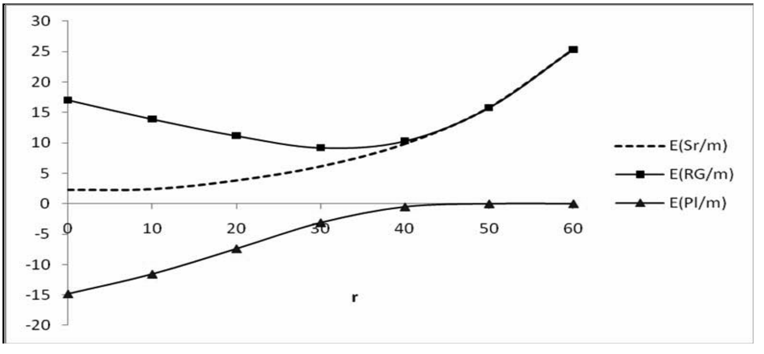

Figure 1. Expected value of retrospective gain, prospective loss and surplus per policy

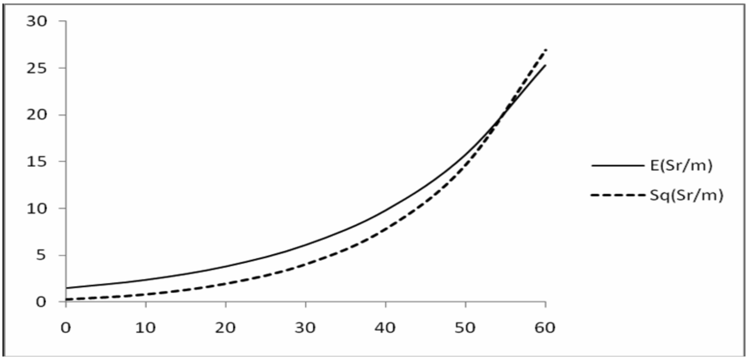

Figure 2. Expected value and standard deviation of surplus per policy

We note that, as the valuation date increases, the standard deviation of the surplus increases. In order to understand this, we have to consider that the standard deviation of the surplus is affected by the uncertainty about the cash flows following the premium and by the variance of the interest rate. When r increases, we have to capitalize a greater number of retrospective cash flows for a longer time and discount a smaller number of prospective cash flows for a shorter period. Consequently, the variance of the capitalized cash gains increases and that of the discounted losses decreases. Numerical investigation shows that the first effect prevails over the second one.

Now we study the expected value of the surplus: it increases as the valuation date increases. We examine this variation by dealing separately with the retrospective gain and the prospective loss.

The expected value of the retrospective gain is affected by two factors: as r increases, the number of cash flows increases and, since they are negative (the only income is the single premium at the inception of the contract), the expected value decreases. At the same time, we have to capitalize the cash flows for a longer period, so that the capitalized value of the premiums and the subsequent cash outflows increases. Overall, as r increases, the net effect is that the expected value of the retrospective gain falls until $r=30$ and rises thereafter. In contrast, as r increases, the prospective loss decreases: and, in fact, we have to consider a smaller number of cash outflows and they have a lower discounted value.

6. Concluding remarks

In this paper we have analyzed the insurance surplus for a portfolio of equity-linked policies. We have adopted a definition of surplus as differences between *Retrospective Gain* and *Prospective Loss* and have derived the first two moments of the surplus distribution. Numerical results have shown the variation of the expected value and variances of the surplus during the life of the contract. The standard deviation can be used as a risk measure in order to evaluate the financial risk of the policies, which increases as the valuation date of the surplus increases.

Further research can drown on this paper and the introduction of stochastic interest rates and surrender options can represent an interesting in depth examination. Moreover, a simulation approach can be developed in order to investigate the whole surplus distribution.

In the light of the model presented and results obtained, we believe that the paper is useful in enhancing an insurer's understanding of the surplus behaviour underlying a basic form of Equity-linked policies. We deem this consideration is important in the perspective of the liquidity and insolvency risk management. With regard to this point, an advantage of the model used is that it allows an ex ante assessment of the insurer's solvency throughout the duration of contract. Consequently, a change to the design of the product can be made, and, in particular, the premium can be modified according to obtain suitable expected value and variance in accordance with solvency requirements.

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