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# A Monte Carlo study on Full Information methods in simultaneous equation models

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*Summary:* The aim of this work is to study - via a Monte Carlo experiment – the small sample behaviour of Full Information Least Orthogonal Distance Estimator and to compare it with other two full information methods, namely Three Stage Least Squares and Full Information Maximum Likelihood. The comparison is made under two distributional assumption about error components: Normal and Uniform distributions. FI LODE appear to have for small samples, a smaller bias than FIML and 3SLS even when the error component is normally distributed. The situation improves when error component is uniformly distributed.

*Keywords:* Simultaneous equations models, Orthogonal distance, Principal Components.

## *1. Introduction*

In a previous simulation experiment (Naccarato, Zurlo, 2008), it has been shown that, apart from some exception, both Limited Information and Full Information LODE feature a lower bias than 2SLS, LI ML and 3SLS estimators.

With respect to MSE, the situation was somehow different LI LODE performs better than 2SLS only for sample size greater than 30, while FI LODE with respect to 3SLS performs quite better than its limited information version for samples of size 20, improving its performance at increasing sample size.

Now a new simulation experiment has been produced mainly to compare FI LODE to FI ML that was not considered in the preceding experiment for computational problems that now have been solved. 3SLS is also considered and compared with.

The new simulation has been performed on the same system of equations and with the same experimental design already considered in the previous one. The only difference is that in this experiment the sample sizes are fixed at 20, 30 and 100, without considering the sample size 50 – that was present in the previous experiment – on the ground that it was not adding relevant information and to obtain a small advantage on the already complex structure of the experiment.

Furthermore the new feature of two distributional assumptions of error component has been introduced. Since Maximum Likelihood estimators are derived under normality assumption for the error component and LODE method is distribution free, Normal and Uniform errors' distribution have been considered. The aim was to see whether the methods of estimation considered were significantly affected by them.

The outline of the paper is the following. After describing the experimental design (§ 2), the computational procedure for FI LODE is briefly outlined (§ 3). The results of the experiment when the distribution of the error component is Normal and when it is Uniform are presented (§ 4). Small sample results are then shown (§ 5) and few words of conclusions end the work (§ 6).

## *2. The design of the experiment*

Using standard notations, a simultaneous equation system can be written as follows:

$$\underset{n,m}{Y}\ \underset{m,m}{\Gamma} + \underset{n,k}{X}\ \underset{k,m}{\mathrm{B}} + \underset{n,m}{U} = \underset{n,m}{0} \qquad (1)$$

where $Y$ is the $n \times m$ matrix of endogenous variables and $\Gamma$ is the corresponding $m \times m$ matrix of structural parameters, $X$ is the $n \times k$ matrix of exogenous variables and $\mathrm{B}$ is the $k \times m$ matrix of their structural parameters. Finally $U$ is the $n \times m$ matrix of disturbances for which standard hypotheses are supposed to hold.

The simulation has been conducted using the three equation model proposed by Cragg in 1967:

$$\begin{cases} y_1 = -0.89 y_2 - 0.16 y_3 + 44 + 0.74 x_2 + 0.13 x_5 \\ y_2 = -0.74 y_1 + 62 + 0.70 x_3 + 0.96 x_5 + 0.06 x_7 \\ y_3 = -0.29 y_2 + 40 + 0.53 x_3 + 0.11 x_4 + 0.56 x_6 \end{cases} \tag{2}$$

In our experiment it is then $n = 20, 30{,}100$ and $m = 3$. Accordingly the structural form parameters matrices are

$$\Gamma = \begin{bmatrix} 1 & -0.89 & -0.16 \\ -0.74 & 1 & 0 \\ 0 & -0.29 & 1 \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} 44 & 62 & 40 \\ 0.74 & 0 & 0 \\ 0 & 0.70 & 0.53 \\ 0 & 0 & 0.11 \\ 0.13 & 0.96 & 0 \\ 0 & 0 & 0.56 \\ 0 & 0.06 & 0 \end{bmatrix} \tag{3}$$

which have to be used to compute the reduced form of the system

$$\underset{n,m}{Y} = \underset{n,k}{X} \, \underset{k,m}{\Pi} + \underset{n,m}{V} \tag{4}$$

where:

$$\underset{k,m}{\Pi} = -\underset{k,m}{\mathrm{B}}\underset{m,m}{\Gamma^{-1}} = \begin{bmatrix} 353.20 & 323.37 & 133.78 \\ 2.41 & 1.78 & 0.52 \\ 2.41 & 2.48 & 1.25 \\ 0.06 & 0.04 & 0.12 \\ 3.35 & 3.44 & 0.99 \\ 0.29 & 0.21 & 0.62 \\ 0.18 & 0.19 & 0.06 \end{bmatrix} \tag{5}$$

from this point the generation procedures starts going trough the following three steps.

1. *Exogenous variables generation.* For each sample size exogenous variables are generated from uniform distribution in the following intervals:
   $X_2 = [10-20], X_3 = [15-27], X_4 = [3-12], X_5 = [3-7],$
   $X_6 = [20-50], X_7 = [7-13].$
   Exogenous values are kept constant for each sample size.

2. *Computation of endogenous variables unaffected by error.* Endogenous variables are generated through reduced form equation. Using the following notation for the endogenous variables not affected by error

$$Y^* = X\Pi \tag{6}$$

   where $X$ is the matrix of generated exogenous variables.

3. *Variance covariance matrix of error component generation.* Taking in mind that

$$\underset{n,m}{V} = -\underset{n,m}{U}\underset{m,m}{\Gamma^{-1}} \tag{7}$$

   and that RF variance-covariance matrix

$$\Sigma = \left(\Gamma^{-1}\right)^T \Omega \Gamma^{-1} \tag{8}$$

where $\Omega$ is the variance-covariance matrix of the SF error components. The matrix $\Omega$ has been chosen in the following way:

*a)* *i*ts diagonal elements (i.e the variances of the SF error component) are obtained as a proportion of the variance of $Y\Gamma = Z$ i.e.

$$w_{ii} = s_Z^2 S_i \tag{9}$$

where $S_i$ is a proportionality coefficient chosen randomly from a uniform distribution in three intervals : $[0{,}2 - 0{,}25]$, $[0{,}4 - 0{,}5]$, $[0{,}75 - 0{,}8]$.

*b)* its extra diagonal elements (i.e. the covariances between error components in SF equations) are obtained generating randomly $m(m-1)/2$ correlation coefficients $r_{ij}$ in the following intervals: $[0{,}1 - 0{,}2]$, $[0{,}4 - 0{,}5]$, $[0{,}8 - 0{,}9]$. To each one of them is assigned a random sign. The covariance between error components in equation $i$ and in equation $j$ is computed as

$$w_{ij} = r_{ij}\left(w_{ii}\, w_{jj}\right)^{1/2} \tag{10}$$

Then the matrix $\Sigma$ is obtained according to (8).

4. *Generating error components according to Normal and Uniform distributions.* For each sample of $n$ observations, $m$ series of random numbers are generated independently from a standardized Normal distribution and from a Uniform distribution in the interval $(-\sqrt{3}, \sqrt{3})$ to have zero mean and variance one. According to the spectral decomposition of a non singular variance covariance matrix, the set of contemporaneous dependent error components are obtained multiplying them by the following matrix

$$Q = P\Lambda^{1/2} P^T \quad (11)$$

Where $P$ and $\Lambda$ are respectively the matrix of characteristic vectors and the diagonal one of characteristic roots of (9).

The design of the experiment can be synthesized in the following table

| $S_i$ / $r_{ij}$ | ***0.20-0.25*** | ***0.4-0.5*** | ***0.75-0.80*** |
|---|---|---|---|
| | N=20 | N=20 | N=20 |
| ***0.1-0.2*** | N=30 | N=30 | N=30 |
| | N=100 | N=100 | N=100 |
| | N=20 | N=20 | N=20 |
| ***0.4-0.5*** | N=30 | N=30 | N=30 |
| | N=100 | N=100 | N=100 |
| | N=20 | N=20 | N=20 |
| ***0.8-0.9*** | N=30 | N=30 | N=30 |
| | N=100 | N=100 | N=100 |

The 27 scenarios listed are repeated for Normal and Uniform error components and for each scenario 500 samples have been considered.

To analyze the results of the simulation experiment we have taken into consideration two indicators which represent two relative measures of bias and variability around the parameter value:

- for bias, the following indicator has been considered

$$j = \left(\hat{q} - q\right)/q \quad (12)$$

(i.e. the bias divided by the fixed initial parameter value) where $\hat{q}$ is the average of estimated parameter over the 500 samples and $q$ is one of the $g$ or $b$ parameters;

- for variability

$$y = RMSE/q \tag{13}$$

where RMSE is the Root Mean Square Error of $\hat{q}$ which is divided by the initial parameter value.

The use of relative measures has been made to facilitate comparison among estimates of different parameters.

### *3. The computational procedure*

While to obtain 3SLS and FI ML estimates standard computational procedures have been used, it is – may be – useful to spend few words about the one used for FI LODE.

The latter are obtained (see Naccarato, 2007, p. 97) minimizing the quadratic form:

$$\underset{1,S}{d^T}\ \underset{S,mk}{\hat{\Pi}_*^T}\ \underset{mk,mk}{\left(\Omega \otimes \left(X^T X\right)^{-1}\right)^{-1}}\ \underset{mk,S}{\hat{\Pi}_*}\ \underset{S,1}{d} = d^T \hat{\Pi}_*^T \left(\Omega^{-1} \otimes \left(X^T X\right)\right) \hat{\Pi}_* d \tag{14}$$

with respect to $\delta$ which is the vector of unknown parameters of SF equations; where $\Omega$ is the variance-covariance matrix of its error components, $\hat{\Pi}_*$ is the reduced form parameters matrix and $X$ is the exogenous variables' one. The minimization of (14) is obtained trough those vectors which are associated with the $m$ smaller characteristic roots of the matrix

$$\underset{S,S}{A} = \hat{\Pi}_*^T \left(\Omega^{-1} \otimes \left(X^T X\right)\right)\hat{\Pi}_* =$$

$$= \begin{bmatrix} \hat{\Pi}_*^{1T}\sigma^{11}\left(X^T X\right)^{-1}\hat{\Pi}_*^{1} & \cdots & \hat{\Pi}_*^{1T}\sigma^{1i}\left(X^T X\right)^{-1}\hat{\Pi}_*^{i} & \cdots & \hat{\Pi}_*^{1T}\sigma^{1m}\left(X^T X\right)^{-1}\hat{\Pi}_*^{m} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \hat{\Pi}_*^{iT}\sigma^{i1}\left(X^T X\right)^{-1}\hat{\Pi}_*^{1} & \cdots & \hat{\Pi}_*^{iT}\sigma^{ii}\left(X^T X\right)^{-1}\hat{\Pi}_*^{i} & \cdots & \hat{\Pi}_*^{iT}\sigma^{im}\left(X^T X\right)^{-1}\hat{\Pi}_*^{m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{\Pi}_*^{mT}\sigma^{m1}\left(X^T X\right)^{-1}\hat{\Pi}_*^{1} & \cdots & \hat{\Pi}_*^{mT}\sigma^{mi}\left(X^T X\right)^{-1}\hat{\Pi}_*^{i} & \cdots & \hat{\Pi}_*^{mT}\sigma^{mm}\left(X^T X\right)^{-1}\hat{\Pi}_*^{m} \end{bmatrix} \tag{15}$$

Notice that to minimize the quadratic form (14) means to minimize the trace of the sample estimate of the matrix $\Omega$ i.e. $\sum_{i=1}^{m} \hat{s}_i^2$ In other words vector $\hat{d}_{LODE}$ gives rise to a matrix $\hat{\Omega}$ such that

$$tr\left(\hat{\Omega}\right) = \sum_{i=1}^{m} \hat{s}_i^2 = \min \tag{16}$$

It has to be stressed that the minimization of $tr\left(\hat{\Omega}\right)$ does not imply the minimization of each term of the sum, i.e. of every residual variance $\hat{s}_i^2$ of the *m* equations.

With this in mind, the computational procedure for FI LODE that has been developed goes along the following lines of reasoning.

Let us assume – for the moment – that the error components are uncorrelated between equations (in this case the $A$ matrix is block-diagonal) and let $l_{A\min}$ be its smallest characteristic root. Let $l_{A_{11}\min} \ldots l_{A_{mm}\min}$ be the set of the smallest characteristic root of the *m* block diagonal matrices $A_{ii}$. It has to be

$$l_{A\min} = \min\left\{l_{A_{11}\min}, \ldots, l_{A_{mm}\min}\right\} \tag{17}$$

and it has to be noticed that it is not known “a priori” to which one of the smallest characteristic roots (i.e. to which one of the block diagonal matrix) it corresponds.
Furthermore in this situation the characteristic vectors have non zero elements only in correspondence to the block diagonal matrix to which they refer The FI LODE then reduces to the LI one if all the $m$ smallest characteristic roots of the block diagonal matrices and their associated vectors are taken into account simultaneously.
In the usual case of correlated disturbances the matrix $A$ is no more block diagonal. In analogy to the preceding case, the first $m$ smaller characteristic roots and their associated vectors have been taken into account in the computational procedure. The characteristic vectors associated to the $m$ smaller characteristic roots of matrix $A$ are partitioned according to each equation. The sub-vector, among the $m$, which minimizes the residual variance gives, after normalization, the FI LODE of each equation’s structural parameters. The reordered set of minimizing sub-vectors gives the vector of estimates.

## *4. Results of the experiment*

To synthesize results of the simulation experiment, the percentage of times in which parameters’ estimators present the lowest bias or RMSE among the three estimation method has been considered both for Normal and Uniform distribution.

*a) Normal error component*

First let us consider the case in which the error component is distributed according to a Multivariate Normal $(0,\Sigma)$, where $\Sigma$ is defined in (8) and the error component with that variance is obtained through (11) starting from the generation of normally standardized independent random numbers.

For small sample sizes most of time LODE estimator outperforms 3SLS estimators in terms of bias, while FIML estimator display the best results (see Table 1; all tables are reported in Appendix). Note that when the correlation coefficient between the disturbances displays small values (0,1-0,2) and for all the variance values ($S_i$) considered in the experiment, LODE estimator shows similar performances of FIML. For increasing values of the correlation coefficient this result becomes weaker, that is the frequency of better results of LODE estimator decrease when the correlation between the error components increases.

In terms of RMSE, the estimates obtained with FIML and 3SLS estimators show more frequently lower values than LODE method (see Table 2).

Notice that when LODE is compared only to 3SLS estimators – as it was in the previous simulation experiment (Naccarato, Zurlo, 2008) – LODE estimators display lower bias than 3SLS almost for all simulation conditions and for all sample sizes considered, confirming in this way the results already obtained. Also with regard to RMSE to comparison between LODE and 3SLS estimates confirms the results already obtained: LODE estimator presents greater frequencies of estimates with lower RMSE than 3SLS in most of cases under analysis.

Since 3SLS estimation is generally preferred to FIML, because the latter has sometimes computational problems, it is worthwhile to stress the point. Furthermore normality assumption for the error component is often not practical.

*b) Uniform error component*

In order to obtain results comparable with normally distributed error components, a second simulation experiment has been carried out using the Uniform distribution in $\left(-\sqrt{3}, \sqrt{3}\right)$.

About the bias of the estimators it has to be notice that LODE estimator shows lower bias than 3SLS and FIML more frequently than the results obtained under Normality condition (see Table 3). This is particularly true for the scenarios related to small sample sizes. Similarly to what has been seen previously, when the correlation coefficient between the disturbances increases FIML estimator presents more frequently estimates affected by lower bias.

The comparison in terms of RMSE (see Table 4) shows that FIML estimators are still to be preferred since the number of times they produce estimates with lower RMSE is very high for all the scenarios considered.

Considering that standardized Uniform distribution has a very short range of variation, in order to evaluate more deeply the effect of more scattered errors components a second Uniform distribution has been considered in the interval $(-10, 10)$. In point of fact in this situation, LODE estimator performs better than FIML in terms of both bias and RMSE.

These results represent an improvements with respect to the previous uniform distribution; the bias of LODE estimators are largely better than FIML estimators (see Table 5). In point of fact LODE estimators perform better than the others in terms of bias, in most of the scenarios. This happens more frequently when dealing with small sample. Moreover, it has to be noticed that – differently from the previous two cases considered – the results related to LODE estimators do not seem to be affected by the correlation between the error components as on the contrary it is for the other two methods.

As far as RMSE of estimators are concerned (see Table 6), when the disturbances are uniformly distributed in $(-10, 10)$ the comparison has to be made only between LODE and 3SLS, since every time FIML estimators produce higher RMSE than the other two methods. 3SLS estimation mostly presents a lower RMSE, with the exception of some cases in which LODE outperforms it.

## *5. Small sample analysis*

As it has been pointed out in the previous paragraph, LODE method seems to work particularly well when dealing with small samples. To go in greater details about this point, let us now examine the case of sample size equal to twenty.

To this extent the average of relative Bias and RMSE over the whole set of parameters have been computed for each one of the proposed scenarios and for each one of the distributional assumptions on error component.

In Table 7 (Appendix 1) the computed values for random normally generated errors are shown and it can be seen that LODE bias is almost every time lower than those of 3SLS and FIML. Frequently it gives raise to a bias reduction greater than or equal to 20%. The relative bias average seems to be very little affected by increasing variance and correlation.

As far as RMSE is concerned the method which seems to present lower values than the others is the 3SLS even if LODE performance is not at all bad. FIML appears to be strongly affected by increasing correlation of the error components.

In Table 8 (Appendix 1) are shown the same averages but for random errors uniformly generated in the interval $(-\sqrt{3}, \sqrt{3})$. The average bias presented by the three methods seems to be greater than those of the Normal case. 3SLS presents every time the highest value and FIML works generally better than LODE.

RMSE are strongly increased with respect to the Normal situation and FIML estimators present the lower values with respect to 3SLS, which represents the second best, and LODE which is the worst.

The performance of the three methods changes completely when considering error generated from a Uniform distribution in (-10, 10) (see Table 9). Here the method that presents lower bias is mostly LODE while FIML is the one that has higher bias.

With respect to RMSE 3SLS is the best and LODE the second one even if the difference between the two methods are not so great. The one that is performing really bad is FIML that presents values which are frequently more than three times that of 3SLS.

To better illustrate the point about small sample performance of LODE method, let us consider the case of the three equations system for the first scenario and sample size equal to 20. Since from the preceding analysis it is evident some kind of uniformity among scenarios the first one can be considered an illustrative example of what happens in the most part of the others.

To this extent box plots of the distributions of parameters for the three equations system with Normal, Uniform in $(-\sqrt{3}, \sqrt{3})$ and Uniform in (-10, 10) distributions are shown in graph 1 – 3 (Appendix 2). Notice that with regard to the last distribution, an “ad hoc” scale has been adopted for graphical representation because of the high variability presented by the estimators. It has furthermore to be taken in mind that in the graphical representation adopted, the median is represented by a line within the box while the mean by a black square; the horizontal sides of the box are 1st and 3rd quartiles respectively.

With regard to Normal and Uniform in $(-\sqrt{3},\sqrt{3})$ distributions of error components it has to be noticed that the empirical sample distribution of 3SLS estimators are the ones with greater bias even if sometimes with smaller variability. The other two methods give rise to distributions with a very similar bias except for some cases (equation 1) in which FIML produces more biased estimators. It has to be notice that the asymmetry index Table 16 (Appendix 1) are for both LODE and

FIML methods (but also for 3SLS) not too much different from zero except for FIML in the first equation. For the same equation the kurtosis index presents values exceedingly high for the empirical distribution of FIML estimators of. This situation seems to be verified also in the other scenario with sample size equal to 20.

The estimators' behavior of the three equations can be more precisely seen in Tables 13-15 (Appendix 1) where the numerical values of Bias and RMSE are presented.

## *6. Conclusions*

Following a previous simulation experiment mainly devoted to the study of Limited Information methods the present paper concentrates on Full Information ones. FILODE, 3SLS and FIML are compared through a wide Monte Carlo study in which 9 different scenarios have been considered according to error components' variance and correlation. For each scenario three sample sizes (20, 30 and 100) have been considered.

Furthermore two distributional hypotheses about disturbances, Normal and Uniform, have been introduced in the new experiment to study the performances of estimation methods with respect to them.

The results of the experiment have not highlighted strong differences between the performances of the three methods as far as the distribution of error component is concerned. Both N (0, 1) and $U(-\sqrt{3},\sqrt{3})$ give almost the same results. A hypothesis of a greater variance Uniform distribution has been then introduced for the generation of error components, namely a Uniform distribution in the interval (-10, 10).

With respect to this last situation a very strong difference among estimation methods has been observed: LODE presents always bias very much lower than FIML; also in comparison of 3SLS the LODE bias is lower. The same happens with RMSE for FIML, while 3SLS seems to have almost every time the lower one.

The most interesting result of the study is the very good performance of LODE in small samples. To show it the case of sample size 20 is more deeply studied.

To this extent a relative Bias and RMSE average over the whole set of the three equations parameters have been considered: LODE Bias is almost every time lower than the one of FIML in particular when the error component is uniformly distributed. With regard to variability 3SLS method presents generally the lowest RMSE, while LODE – even if greater is than – is not very much different. Generally speaking, in small sample size LODE has a lower RMSE than FIML.

For the first scenario of sample size 20, empirical sample distributions of the three equations system parameters' estimators have been considered as illustrative example. Their Box plots show that with regard to Normal and Uniform in $(-\sqrt{3}, \sqrt{3})$ 3SLS estimators are the ones with greater bias even if sometimes with smaller variability, while the other two methods give rise to distributions with a very similar bias except for some cases in which FIML produces more biased estimators.

Box plot for the case of Uniform randomly generated error in the interval (-10, 10), show an evident strong variability of estimators independently by the method used. Notwithstanding that LODE method present mostly a lower bias.

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## *Appendix*

*Table 1. Relative frequency distribution of FILODE, 3SLS and FIML presenting a lower bias grouped by* $S_i$, $r_i$ *and sample size - Normal error component*

| | | $r_i$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.1-0.2 | | | 0.4-0.5 | | | 0.8-0.9 | | |
| $S_i$ | Sample size | LODE | 3SLS | FIML | LODE | 3SLS | FIML | LODE | 3SLS | FIML |
| 0.2-0.25 | 20 | 53.33 | 0.00 | 46.67 | 26.67 | 13.33 | 60.00 | 33.33 | 13.33 | 53.33 |
| 0.4-0.5 | | 73.33 | 6.67 | 20.00 | 46.67 | 13.33 | 40.00 | 73.33 | 0.00 | 26.67 |
| 0.75-0.8 | | 66.67 | 13.33 | 20.00 | 33.33 | 13.33 | 53.33 | 46.67 | 0.00 | 53.33 |
| 0.2-0.25 | 30 | 26.67 | 0.00 | 73.33 | 20.00 | 0.00 | 80.00 | 13.33 | 0.00 | 86.67 |
| 0.4-0.5 | | 26.67 | 0.00 | 73.33 | 20.00 | 0.00 | 80.00 | 53.33 | 20.00 | 26.67 |
| 0.75-0.8 | | 26.67 | 0.00 | 73.33 | 46.67 | 0.00 | 53.33 | 46.67 | 0.00 | 53.33 |
| 0.2-0.25 | 100 | 6.67 | 0.00 | 93.33 | 6.67 | 6.67 | 86.67 | 26.67 | 0.00 | 73.33 |
| 0.4-0.5 | | 0.00 | 0.00 | 100.00 | 13.33 | 0.00 | 86.67 | 6.67 | 6.67 | 86.67 |
| 0.75-0.8 | | 6.67 | 0.00 | 93.33 | 0.00 | 6.67 | 93.33 | 13.33 | 0.00 | 86.67 |

*Table 2. Relative frequency distribution of FILODE, 3SLS and FIML presenting a lower RMSE grouped by* $S_i$, $r_i$ *and sample size - Normal error component*

| | | $r_i$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.1-0.2 | | | 0.4-0.5 | | | 0.8-0.9 | | |
| $S_i$ | Sample size | LODE | 3SLS | FIML | LODE | 3SLS | FIML | LODE | 3SLS | FIML |
| 0.2-0.25 | 20 | 6.67 | 33.33 | 60.00 | 0.00 | 26.67 | 73.33 | 20.00 | 20.00 | 60.00 |
| 0.4-0.5 | | 20.00 | 40.00 | 40.00 | 13.33 | 26.67 | 60.00 | 40.00 | 20.00 | 40.00 |
| 0.75-0.8 | | 26.67 | 46.67 | 26.67 | 13.33 | 20.00 | 66.67 | 6.67 | 26.67 | 66.67 |
| 0.2-0.25 | 30 | 6.67 | 13.33 | 80.00 | 0.00 | 6.67 | 93.33 | 13.33 | 6.67 | 80.00 |
| 0.4-0.5 | | 6.67 | 53.33 | 40.00 | 0.00 | 33.33 | 66.67 | 26.67 | 20.00 | 53.33 |
| 0.75-0.8 | | 40.00 | 33.33 | 26.67 | 46.67 | 40.00 | 13.33 | 20.00 | 33.33 | 46.67 |
| 0.2-0.25 | 100 | 13.33 | 6.67 | 80.00 | 6.67 | 20.00 | 73.33 | 6.67 | 6.67 | 86.67 |
| 0.4-0.5 | | 6.67 | 13.33 | 80.00 | 0.00 | 13.33 | 86.67 | 0.00 | 13.33 | 86.67 |
| 0.75-0.8 | | 20.00 | 6.67 | 73.33 | 13.33 | 26.67 | 60.00 | 6.67 | 20.00 | 73.33 |

*Table 3. Relative frequency distribution of FILODE, 3SLS and FIML presenting a lower bias grouped by* $S_i$, $r_i$ *and sample size - Uniform in (-* $\sqrt{3}$ , $\sqrt{3}$ *) error component*

| | | $r_i$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **0.1-0.2** | | | **0.4-0.5** | | | **0.8-0.9** | | |
| $S_i$ | **Sample size** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | *20* | 53.33 | 6.67 | 40.00 | 33.33 | 20.00 | 46.67 | 73.33 | 13.33 | 13.33 |
| *0.4-0.5* | | 66.67 | 0.00 | 33.33 | 80.00 | 6.67 | 13.33 | 6.67 | 33.33 | 60.00 |
| *0.75-0.8* | | 93.33 | 0.00 | 6.67 | 40.00 | 33.33 | 26.67 | 60.00 | 0.00 | 40.00 |
| *0.2-0.25* | *30* | 53.33 | 0.00 | 46.67 | 53.33 | 0.00 | 46.67 | 0.00 | 0.00 | 100.00 |
| *0.4-0.5* | | 53.33 | 0.00 | 46.67 | 73.33 | 0.00 | 26.67 | 0.00 | 0.00 | 100.00 |
| *0.75-0.8* | | 53.33 | 6.67 | 40.00 | 20.00 | 0.00 | 80.00 | 20.00 | 0.00 | 80.00 |
| *0.2-0.25* | *100* | 20.00 | 0.00 | 80.00 | 40.00 | 26.67 | 33.33 | 13.33 | 13.33 | 73.33 |
| *0.4-0.5* | | 26.67 | 0.00 | 73.33 | 6.67 | 0.00 | 93.33 | 6.67 | 0.00 | 93.33 |
| *0.75-0.8* | | 60.00 | 6.67 | 33.33 | 13.33 | 0.00 | 86.67 | 13.33 | 0.00 | 86.67 |

*Table 4. Relative frequency distribution of FILODE, 3SLS and FIML presenting a lower RMSE grouped by* $S_i$, $r_i$ *and sample size - Uniform in (-* $\sqrt{3}$ , $\sqrt{3}$ *) error component*

| | | $r_i$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **0.1-0.2** | | | **0.4-0.5** | | | **0.8-0.9** | | |
| $S_i$ | **Sample size** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | *20* | 20.00 | 33.33 | 46.67 | 26.67 | 13.33 | 60.00 | 6.67 | 13.33 | 86.67 |
| *0.4-0.5* | | 6.67 | 80.00 | 13.33 | 6.67 | 20.00 | 73.33 | 0.00 | 13.33 | 86.67 |
| *0.75-0.8* | | 6.67 | 60.00 | 33.33 | 13.33 | 46.67 | 40.00 | 26.67 | 26.67 | 46.67 |
| *0.2-0.25* | *30* | 40.00 | 13.33 | 46.67 | 13.33 | 26.67 | 60.00 | 0.00 | 13.33 | 86.67 |
| *0.4-0.5* | | 6.67 | 20.00 | 73.33 | 20.00 | 13.33 | 66.67 | 0.00 | 26.67 | 73.33 |
| *0.75-0.8* | | 6.67 | 60.00 | 33.33 | 0.00 | 26.67 | 73.33 | 13.33 | 20.00 | 66.67 |
| *0.2-0.25* | *100* | 40.00 | 13.33 | 46.67 | 26.67 | 20.00 | 53.33 | 6.67 | 26.67 | 66.67 |
| *0.4-0.5* | | 0.00 | 13.33 | 86.67 | 6.67 | 20.00 | 73.33 | 6.67 | 13.33 | 80.00 |
| *0.75-0.8* | | 0.00 | 6.67 | 93.33 | 0.00 | 26.67 | 73.33 | 6.67 | 6.67 | 86.67 |

*Table 5. Relative frequency distribution of FILODE, 3SLS and FIML presenting a lower bias grouped by* $S_i$, $r_i$ *and sample size - Uniform in (-10, 10) error component*

| | | $r_i$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **0.1-0.2** | | | **0.4-0.5** | | | **0.8-0.9** | | |
| $S_i$ | **Sample size** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | *20* | 60.00 | 13.33 | 26.67 | 40.00 | 13.33 | 46.67 | 40.00 | 6.67 | 53.33 |
| *0.4-0.5* | | 40.00 | 13.33 | 46.67 | 60.00 | 6.67 | 33.33 | 60.00 | 13.33 | 26.67 |
| *0.75-0.8* | | 40.00 | 40.00 | 20.00 | 33.33 | 20.00 | 46.67 | 46.67 | 13.33 | 40.00 |
| *0.2-0.25* | *30* | 86.67 | 13.33 | 0.00 | 73.33 | 13.33 | 13.33 | 53.33 | 13.33 | 33.33 |
| *0.4-0.5* | | 46.67 | 33.33 | 20.00 | 60.00 | 13.33 | 26.67 | 66.67 | 13.33 | 20.00 |
| *0.75-0.8* | | 66.67 | 33.33 | 0.00 | 46.67 | 20.00 | 33.33 | 60.00 | 26.67 | 13.33 |
| *0.2-0.25* | *100* | 80.00 | 13.33 | 6.67 | 73.33 | 20.00 | 6.67 | 46.67 | 6.67 | 46.67 |
| *0.4-0.5* | | 80.00 | 20.00 | 0.00 | 13.33 | 86.67 | 0.00 | 66.67 | 6.67 | 26.67 |
| *0.75-0.8* | | 80.00 | 20.00 | 0.00 | 26.67 | 0.00 | 73.33 | 86.67 | 13.33 | 0.00 |

*Table 6. Relative frequency distribution of FILODE, 3SLS and FIML presenting a lower RMSE grouped by* $S_i$, $r_i$ *and sample size - Uniform in (-10, 10) error component*

| | | $r_i$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **0.1-0.2** | | | **0.4-0.5** | | | **0.8-0.9** | | |
| $S_i$ | **Sample size** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | *20* | 40.00 | 60.00 | 0.00 | 26.67 | 66.67 | 6.67 | 53.33 | 46.67 | 6.67 |
| *0.4-0.5* | | 53.33 | 46.67 | 0.00 | 20.00 | 73.33 | 6.67 | 20.00 | 80.00 | 0.00 |
| *0.75-0.8* | | 53.33 | 46.67 | 0.00 | 46.67 | 53.33 | 0.00 | 46.67 | 53.33 | 0.00 |
| *0.2-0.25* | *30* | 46.67 | 53.33 | 0.00 | 13.33 | 86.67 | 0.00 | 40.00 | 60.00 | 0.00 |
| *0.4-0.5* | | 60.00 | 40.00 | 0.00 | 6.67 | 93.33 | 0.00 | 40.00 | 60.00 | 0.00 |
| *0.75-0.8* | | 53.33 | 40.00 | 6.67 | 53.33 | 46.67 | 0.00 | 20.00 | 80.00 | 0.00 |
| *0.2-0.25* | *100* | 40.00 | 60.00 | 0.00 | 20.00 | 80.00 | 0.00 | 40.00 | 40.00 | 20.00 |
| *0.4-0.5* | | 6.67 | 93.33 | 0.00 | 33.33 | 66.67 | 0.00 | 53.33 | 46.67 | 0.00 |
| *0.75-0.8* | | 26.67 | 73.33 | 0.00 | 13.33 | 86.67 | 0.00 | 46.67 | 53.33 | 0.00 |

*Table 7. Average Bias and RMSE, Normal distribution, sample size 20*

| Bias | | | | | RMSE | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** | $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.1-0.2** | 0.14 | 0.41 | 0.10 | *0.2-0.25* | **0.1-0.2** | 1.40 | 1.29 | 1.32 |
| *0.4-0.5* | | 0.08 | 0.34 | 0.59 | *0.4-0.5* | | 1.11 | 1.40 | 3.96 |
| *0.75-0.80* | | 0.20 | 0.48 | 0.25 | *0.75-0.80* | | 1.62 | 1.98 | 2.50 |
| | | | | | | | | | |
| | | **LODE** | **3SLS** | **FIML** | | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.4-0.5** | 0.19 | 0.32 | 0.22 | *0.2-0.25* | **0.4-0.5** | 1.24 | 1.14 | 0.95 |
| *0.4-0.5* | | 0.13 | 0.54 | 0.21 | *0.4-0.5* | | 1.50 | 1.40 | 2.53 |
| *0.75-0.80* | | 0.27 | 0.77 | 0.59 | *0.75-0.80* | | 1.35 | 2.11 | 1.81 |
| | | | | | | | | | |
| | | **LODE** | **3SLS** | **FIML** | | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.8-0.9** | 0.09 | 0.22 | 0.07 | *0.2-0.25* | **0.8-0.9** | 0.90 | 0.99 | 0.96 |
| *0.4-0.5* | | 0.25 | 0.57 | 3.35 | *0.4-0.5* | | 0.75 | 1.16 | 4.98 |
| *0.75-0.80* | | 0.51 | 0.93 | 2.34 | *0.75-0.80* | | 1.91 | 1.38 | 28.43 |

*Table 8. Average Bias and RMSE, Uniform distribution in* ($-\sqrt{3}$ , $\sqrt{3}$ )*, sample size 20*

| Bias | | | | | RMSE | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** | $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.1-0.2** | 0.09 | 0.24 | 0.05 | *0.2-0.25* | **0.1-0.2** | 1.17 | 1.06 | 0.95 |
| *0.4-0.5* | | 0.06 | 0.39 | 0.06 | *0.4-0.5* | | 1.69 | 1.27 | 1.31 |
| *0.75-0.80* | | 0.08 | 0.52 | 0.13 | *0.75-0.80* | | 2.78 | 1.83 | 1.76 |
| | | | | | | | | | |
| | | **LODE** | **3SLS** | **FIML** | | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.4-0.5** | 0.12 | 0.23 | 0.04 | *0.2-0.25* | **0.4-0.5** | 1.28 | 1.16 | 0.85 |
| *0.4-0.5* | | 0.06 | 0.37 | 0.09 | *0.4-0.5* | | 1.46 | 1.19 | 0.85 |
| *0.75-0.80* | | 0.24 | 0.99 | 0.12 | *0.75-0.80* | | 3.03 | 2.27 | 1.86 |
| | | | | | | | | | |
| | | **LODE** | **3SLS** | **FIML** | | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.8-0.9** | 0.08 | 0.41 | 0.11 | *0.2-0.25* | **0.8-0.9** | 1.05 | 1.07 | 0.63 |
| *0.4-0.5* | | 0.26 | 0.23 | 0.07 | *0.4-0.5* | | 1.55 | 1.31 | 0.75 |
| *0.75-0.80* | | 0.31 | 0.99 | 0.23 | *0.75-0.80* | | 2.35 | 1.93 | 1.63 |

*Table 9. Average Bias and RMSE, Uniform distribution in (-10, 10), sample size 20*

| Bias | | | | |
|---|---|---|---|---|
| $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.1-0.2** | 0.43 | 0.86 | 1.18 |
| *0.4-0.5* | | 0.83 | 0.88 | 1.38 |
| *0.75-0.80* | | 0.85 | 0.72 | 4.52 |
| | | | | |
| | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.4-0.5** | 0.62 | 0.70 | 0.95 |
| *0.4-0.5* | | 1.62 | 2.46 | 1.85 |
| *0.75-0.80* | | 0.97 | 1.25 | 2.63 |
| | | | | |
| | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.8-0.9** | 1.03 | 1.12 | 0.71 |
| *0.4-0.5* | | 1.35 | 1.46 | 1.37 |
| *0.75-0.80* | | 1.56 | 1.53 | 1.59 |

| RMSE | | | | |
|---|---|---|---|---|
| $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.1-0.2** | 3.60 | 4.60 | 26.31 |
| *0.4-0.5* | | 7.43 | 6.63 | 40.10 |
| *0.75-0.80* | | 12.80 | 9.31 | 91.69 |
| | | | | |
| | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.4-0.5** | 4.46 | 3.83 | 9.83 |
| *0.4-0.5* | | 9.96 | 5.82 | 13.98 |
| *0.75-0.80* | | 11.03 | 6.97 | 53.27 |
| | | | | |
| | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.8-0.9** | 3.75 | 2.36 | 10.30 |
| *0.4-0.5* | | 11.46 | 8.68 | 25.67 |
| *0.75-0.80* | | 6.52 | 3.54 | 33.29 |

*Table 10. Average Bias and RMSE, Normal distribution, sample size 100*

| Bias | | | | |
|---|---|---|---|---|
| $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.1-0.2** | 0.04 | 0.29 | 0.01 |
| *0.4-0.5* | | 0.08 | 0.32 | 0.02 |
| *0.75-0.80* | | 0.11 | 0.47 | 0.01 |
| | | | | |
| | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.4-0.5** | 0.04 | 0.16 | 0.01 |
| *0.4-0.5* | | 0.32 | 0.52 | 0.21 |
| *0.75-0.80* | | 0.14 | 0.44 | 0.02 |
| | | | | |
| | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.8-0.9** | 0.32 | 0.55 | 0.21 |
| *0.4-0.5* | | 0.12 | 0.23 | 0.01 |
| *0.75-0.80* | | 0.20 | 0.70 | 0.04 |

| RMSE | | | | |
|---|---|---|---|---|
| $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.1-0.2** | 0.49 | 0.55 | 0.43 |
| *0.4-0.5* | | 0.72 | 0.64 | 0.51 |
| *0.75-0.80* | | 0.81 | 0.80 | 0.69 |
| | | | | |
| | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.4-0.5** | 0.46 | 0.46 | 0.35 |
| *0.4-0.5* | | 0.62 | 0.64 | 0.53 |
| *0.75-0.80* | | 0.65 | 0.79 | 0.55 |
| | | | | |
| | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.8-0.9** | 0.57 | 0.66 | 0.44 |
| *0.4-0.5* | | 0.56 | 0.62 | 0.38 |
| *0.75-0.80* | | 1.07 | 0.92 | 0.51 |

*Table 11. Average Bias and RMSE, Uniform distribution in (-* $\sqrt{3}$ *,* $\sqrt{3}$ *), sample size 100*

| Bias | | | | | RMSE | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** | $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.1-0.2** | 0.04 | 0.19 | 0.01 | *0.2-0.25* | **0.1-0.2** | 0.45 | 0.46 | 0.41 |
| *0.4-0.5* | | 0.09 | 0.41 | 0.02 | *0.4-0.5* | | 0.90 | 0.66 | 0.55 |
| *0.75-0.80* | | 0.11 | 0.48 | 0.03 | *0.75-0.80* | | 1.11 | 0.82 | 0.68 |
| | | | | | | | | | |
| | | **LODE** | **3SLS** | **FIML** | | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.4-0.5** | 0.05 | 0.09 | 0.01 | *0.2-0.25* | **0.4-0.5** | 0.44 | 0.44 | 0.39 |
| *0.4-0.5* | | 0.08 | 0.34 | 0.02 | *0.4-0.5* | | 0.76 | 0.68 | 0.44 |
| *0.75-0.80* | | 0.08 | 0.38 | 0.02 | *0.75-0.80* | | 0.88 | 0.70 | 0.59 |
| | | | | | | | | | |
| | | **LODE** | **3SLS** | **FIML** | | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.8-0.9** | 0.04 | 0.22 | 0.01 | *0.2-0.25* | **0.8-0.9** | 0.47 | 0.53 | 0.34 |
| *0.4-0.5* | | 0.08 | 0.41 | 0.01 | *0.4-0.5* | | 0.66 | 0.65 | 0.38 |
| *0.75-0.80* | | 0.24 | 0.82 | 0.06 | *0.75-0.80* | | 1.17 | 1.01 | 0.63 |

*Table 12. Average Bias and RMSE, Uniform distribution in (-10, 10), sample size 100*

| Bias | | | | | RMSE | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** | $S_i$ | $r_i$ | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.1-0.2** | 0.50 | 0.69 | 100.16 | *0.2-0.25* | **0.1-0.2** | 1.09 | 1.14 | 1321.70 |
| *0.4-0.5* | | 0.73 | 0.81 | 210.67 | *0.4-0.5* | | 6.15 | 2.12 | 6265.56 |
| *0.75-0.80* | | 0.52 | 0.77 | 1050.24 | *0.75-0.80* | | 7.07 | 2.45 | 10202.08 |
| | | | | | | | | | |
| | | **LODE** | **3SLS** | **FIML** | | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.4-0.5** | 0.76 | 0.76 | 168.46 | *0.2-0.25* | **0.4-0.5** | 3.48 | 1.49 | 2877.42 |
| *0.4-0.5* | | 0.75 | 0.85 | 315.98 | *0.4-0.5* | | 6.12 | 1.92 | 3007.15 |
| *0.75-0.80* | | 1.05 | 1.63 | 255.60 | *0.75-0.80* | | 6.88 | 3.02 | 2900.57 |
| | | | | | | | | | |
| | | **LODE** | **3SLS** | **FIML** | | | **LODE** | **3SLS** | **FIML** |
| *0.2-0.25* | **0.8-0.9** | 0.63 | 1.13 | 55.91 | *0.2-0.25* | **0.8-0.9** | 3.11 | 1.48 | 479.65 |
| *0.4-0.5* | | 0.90 | 1.17 | 284.81 | *0.4-0.5* | | 3.23 | 1.68 | 1872.06 |
| *0.75-0.80* | | 0.77 | 0.97 | 331.92 | *0.75-0.80* | | 4.39 | 1.67 | 3482.81 |

*Table 13. Bias and RMSE,* $S_i \in (0{,}2-0{,}25)$ *and* $r_i \in (0{,}1-0{,}2)$*, sample size 20, Normal distribution.*

| | **Bias** | | | **RMSE** | | |
|---|---|---|---|---|---|---|
| **Parameters** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *equation 1* | | | | | | |
| -0,89 | -0.010 | 0.032 | -0.001 | 0.992 | 1.032 | 1.014 |
| -0,16 | -0.009 | -0.150 | -0.159 | 1.044 | 0.894 | 1.812 |
| 44 | 0.076 | -0.136 | 0.100 | 0.298 | 0.250 | 0.857 |
| 0,74 | 0.026 | -0.068 | 0.048 | 0.180 | 0.157 | 0.370 |
| 0,13 | 0.389 | -1.027 | 0.611 | 2.553 | 2.193 | 5.092 |
| *equation 2* | | | | | | |
| -0,74 | 0.009 | 0.097 | -0.009 | 0.108 | 0.121 | 0.095 |
| 62 | -0.035 | -0.427 | 0.041 | 0.490 | 0.505 | 0.401 |
| 0,70 | -0.041 | -0.315 | 0.015 | 0.378 | 0.399 | 0.339 |
| 0,96 | 0.024 | -0.289 | 0.033 | 0.468 | 0.502 | 0.441 |
| 0,06 | -0.583 | -2.139 | 0.320 | 5.361 | 4.239 | 4.738 |
| *equation 3* | | | | | | |
| -0,29 | -0.072 | 0.170 | 0.016 | 0.745 | 0.711 | 0.407 |
| 40 | 0.183 | -0.447 | -0.024 | 1.991 | 1.899 | 1.054 |
| 0,53 | 0.145 | -0.279 | -0.042 | 1.324 | 1.202 | 0.809 |
| 0,11 | 0.445 | -0.589 | -0.033 | 4.780 | 4.920 | 2.137 |
| 0,56 | -0.011 | 0.012 | -0.025 | 0.230 | 0.255 | 0.201 |

*Table 14. Bias and RMSE,* $S_i \in (0,2-0,25)$ *and* $r_i \in (0,1-0,2)$*, sample size 20, Uniform distribution in* $(-\sqrt{3}, \sqrt{3})$ *.*

| | **Bias** | | | **RMSE** | | |
|---|---|---|---|---|---|---|
| **Parameters** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *equation 1* | | | | | | |
| -0,89 | -0.010 | 0.028 | -0.002 | 0.991 | 1.029 | 0.999 |
| -0,16 | 0.010 | -0.059 | 0.012 | 1.053 | 0.978 | 1.070 |
| 44 | 0.007 | -0.179 | 0.011 | 0.228 | 0.261 | 0.236 |
| 0,74 | 0.024 | -0.045 | 0.003 | 0.134 | 0.126 | 0.129 |
| 0,13 | 0.136 | -0.635 | 0.018 | 1.871 | 1.707 | 1.814 |
| *equation 2* | | | | | | |
| -0,74 | 0.004 | 0.055 | -0.005 | 0.065 | 0.075 | 0.067 |
| 62 | -0.026 | -0.287 | 0.030 | 0.426 | 0.382 | 0.358 |
| 0,70 | -0.008 | -0.131 | 0.011 | 0.197 | 0.220 | 0.183 |
| 0,96 | -0.002 | -0.062 | -0.024 | 0.352 | 0.309 | 0.324 |
| 0,06 | 0.221 | 1.571 | -0.354 | 5.806 | 4.376 | 5.106 |
| *equation 3* | | | | | | |
| -0,29 | -0.202 | 0.093 | -0.053 | 0.633 | 0.702 | 0.370 |
| 40 | 0.555 | -0.223 | 0.145 | 1.701 | 1.803 | 0.984 |
| 0,53 | 0.022 | -0.250 | 0.060 | 0.923 | 1.552 | 0.614 |
| 0,11 | -0.065 | -0.037 | 0.078 | 3.000 | 2.239 | 1.811 |
| 0,56 | 0.036 | 0.007 | 0.004 | 0.206 | 0.182 | 0.148 |

*Table 15. Bias and RMSE,* $S_i \in (0{,}2 - 0{,}25)$ *and* $r_i \in (0{,}1 - 0{,}2)$*, sample size 20, Uniform distribution in (-10, 10).*

| | **Bias** | | | **RMSE** | | |
|---|---|---|---|---|---|---|
| **Parameters** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *equation 1* | | | | | | |
| -0,89 | 0.097 | 0.153 | -0.323 | 1.147 | 1.155 | 2.532 |
| -0,16 | -0.128 | -0.221 | 3.551 | 2.972 | 1.069 | 33.452 |
| 44 | -0.662 | -0.976 | -0.132 | 1.917 | 1.034 | 19.417 |
| 0,74 | -0.294 | -0.509 | 1.068 | 1.602 | 0.752 | 10.364 |
| 0,13 | -0.669 | -3.752 | 7.927 | 2.238 | 9.541 | 141.281 |
| *equation 2* | | | | | | |
| -0,74 | 0.146 | 0.196 | 0.005 | 0.308 | 0.215 | 1.154 |
| 62 | -0.670 | -0.892 | -0.110 | 1.502 | 0.974 | 5.664 |
| 0,70 | -0.454 | -0.649 | 0.257 | 1.999 | 0.939 | 3.919 |
| 0,96 | -0.212 | -0.278 | 0.217 | 2.411 | 1.301 | 5.208 |
| 0,06 | -0.209 | -2.549 | 0.443 | 3.455 | 18.464 | 60.298 |
| *equation 3* | | | | | | |
| -0,29 | 0.448 | 0.487 | -0.541 | 1.379 | 0.864 | 9.053 |
| 40 | -1.092 | -1.367 | 1.228 | 3.688 | 3.882 | 19.428 |
| 0,53 | -1.097 | -0.634 | 1.252 | 4.476 | 8.195 | 22.818 |
| 0,11 | 0.159 | 0.218 | 0.489 | 23.110 | 17.603 | 50.421 |
| 0,56 | -0.087 | -0.028 | 0.168 | 1.723 | 2.942 | 9.672 |

*Table 16. Skewness and Kurtosis,* $S_i \in (0,2-0,25)$ *and* $r_i \in (0,1-0,2)$*, sample size 20, Normal distribution.*

| | **Skewness** | | | **Kurtosis** | | |
|---|---|---|---|---|---|---|
| **Parameters** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *equation 1* | | | | | | |
| -0,89 | 1.33 | -0.23 | 3.27 | 9.53 | 3.13 | 90.04 |
| -0,16 | 0.67 | 0.11 | -8.93 | 5.87 | 2.85 | 99.6 |
| 44 | 0.93 | 0.13 | 12 | 6.8 | 3.5 | 187.4 |
| 0,74 | 0.94 | 0.08 | 9.22 | 8.7 | 3.48 | 132.3 |
| 0,13 | 1.38 | 0.2 | 8.53 | 10.9 | 3.66 | 108.2 |
| *equation 2* | | | | | | |
| -0,74 | -0.43 | -0.8 | 1.35 | 5.15 | 5.43 | 6.16 |
| 62 | 0.45 | 0.72 | 1.32 | 5.15 | 5.46 | 6.04 |
| 0,70 | 0.46 | 0.7 | 1 | 4.7 | 4.24 | 5.69 |
| 0,96 | 0.07 | 0.33 | 0.62 | 5.45 | 3.27 | 4.13 |
| 0,06 | -0.56 | 0.43 | 0.42 | 4.8 | 3.84 | 4.02 |
| *equation 3* | | | | | | |
| -0,29 | -0.23 | -0.05 | 0.34 | 5.8 | 4.5 | 3.68 |
| 40 | 0.28 | 0 | 0.4 | 5.9 | 4.61 | 3.65 |
| 0,53 | 0.28 | 0.15 | 0.27 | 5.66 | 4.58 | 3.77 |
| 0,11 | 1.71 | 0.24 | 0.11 | 12.8 | 3.19 | 3.1 |
| 0,56 | 0.4 | 0.07 | -2.65 | 11.5 | 3.44 | 15.56 |

*Table 17. Skewness and Kurtosis,* $S_i \in (0{,}2 - 0{,}25)$ *and* $r_i \in (0{,}1 - 0{,}2)$*, sample size 20, Uniform distribution in* $(-\sqrt{3}, \sqrt{3})$ *.*

| | **Skewness** | | | **Kurtosis** | | |
|---|---|---|---|---|---|---|
| **Parameters** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *equation 1* | | | | | | |
| -0,89 | 0.45 | -0.34 | -0.43 | 4 | 3.8 | 3.48 |
| -0,16 | 0.33 | -0.03 | 0.2 | 4.5 | 2.7 | 3.13 |
| 44 | 0.52 | 0.6 | 0.44 | 3.8 | 4.8 | 3.3 |
| 0,74 | 0.28 | 0.24 | 0.25 | 3.23 | 3.1 | 3.16 |
| 0,13 | 0.25 | 0.1 | 0.18 | 3.4 | 3 | 3.1 |
| *equation 2* | | | | | | |
| -0,74 | -0.57 | -0.79 | -1 | 4.68 | 4 | 4.8 |
| 62 | 0.5 | 0.8 | 0.9 | 4.7 | 4 | 4.6 |
| 0,70 | 0.6 | 0.5 | 0.75 | 4.07 | 3.55 | 4.33 |
| 0,96 | 0.38 | 0.23 | 0.14 | 3.6 | 2.9 | 3.12 |
| 0,06 | 0 | -0.15 | -0.52 | 4.21 | 2.8 | 4.37 |
| *equation 3* | | | | | | |
| -0,29 | 0.14 | -0.02 | -0.41 | 4.86 | 5.34 | 4.25 |
| 40 | 0.08 | 0.077 | 0.4 | 4.9 | 4.8 | 4.23 |
| 0,53 | -0.14 | -0.13 | 0.28 | 4.3 | 7 | 3.5 |
| 0,11 | -0.8 | -0.23 | -0.11 | 17.7 | 3.3 | 3.27 |
| 0,56 | 0.27 | -0.16 | 0.13 | 6.7 | 2.9 | 3.27 |

*Table 18. Skewness and Kurtosis,* $S_i \in (0{,}2-0{,}25)$ *and* $r_i \in (0{,}1-0{,}2)$*, sample size 20, Uniform distribution in (-10, 10)..*

| | Skewness | | | Kurtosis | | |
|---|---|---|---|---|---|---|
| **Parameters** | **LODE** | **3SLS** | **FIML** | **LODE** | **3SLS** | **FIML** |
| *equation 1* | | | | | | |
| -0,89 | -1.7 | -0.18 | -1.75 | 20.55 | 3.34 | 69 |
| -0,16 | 0.21 | -0.09 | 5.71 | 6.5 | 3 | 81.17 |
| 44 | 1.08 | 0.16 | -4.45 | 28.9 | 3.7 | 67.66 |
| 0,74 | 0.58 | 0.17 | 0.7 | 8.75 | 3.18 | 45.6 |
| 0,13 | 0.5 | 0.1 | 2 | 7.22 | 3 | 54.34 |
| *equation 2* | | | | | | |
| -0,74 | -2 | -0.17 | 1.12 | 21 | 4.36 | 67.45 |
| 62 | 2.73 | 0.28 | -5.07 | 31 | 4.16 | 113.73 |
| 0,70 | 3.52 | 0.16 | 2.77 | 40 | 3.2 | 32.1 |
| 0,96 | -0.14 | -0.01 | 9.62 | 6 | 3 | 173.17 |
| 0,06 | 1.84 | 0.06 | 4.6 | 41.4 | 2.9 | 90.5 |
| *equation 3* | | | | | | |
| -0,29 | 0.51 | -0.011 | -9 | 18.34 | 7.11 | 145 |
| 40 | 1.2 | 0.5 | 6.27 | 24.6 | 6.44 | 92 |
| 0,53 | -0.53 | -0.33 | 8.14 | 11.54 | 3.54 | 117 |
| 0,11 | 0.17 | 0.15 | 2 | 8.38 | 3.13 | 115.8 |
| 0,56 | -1.28 | -0.23 | 16.43 | 14.7 | 3.6 | 339.8 |

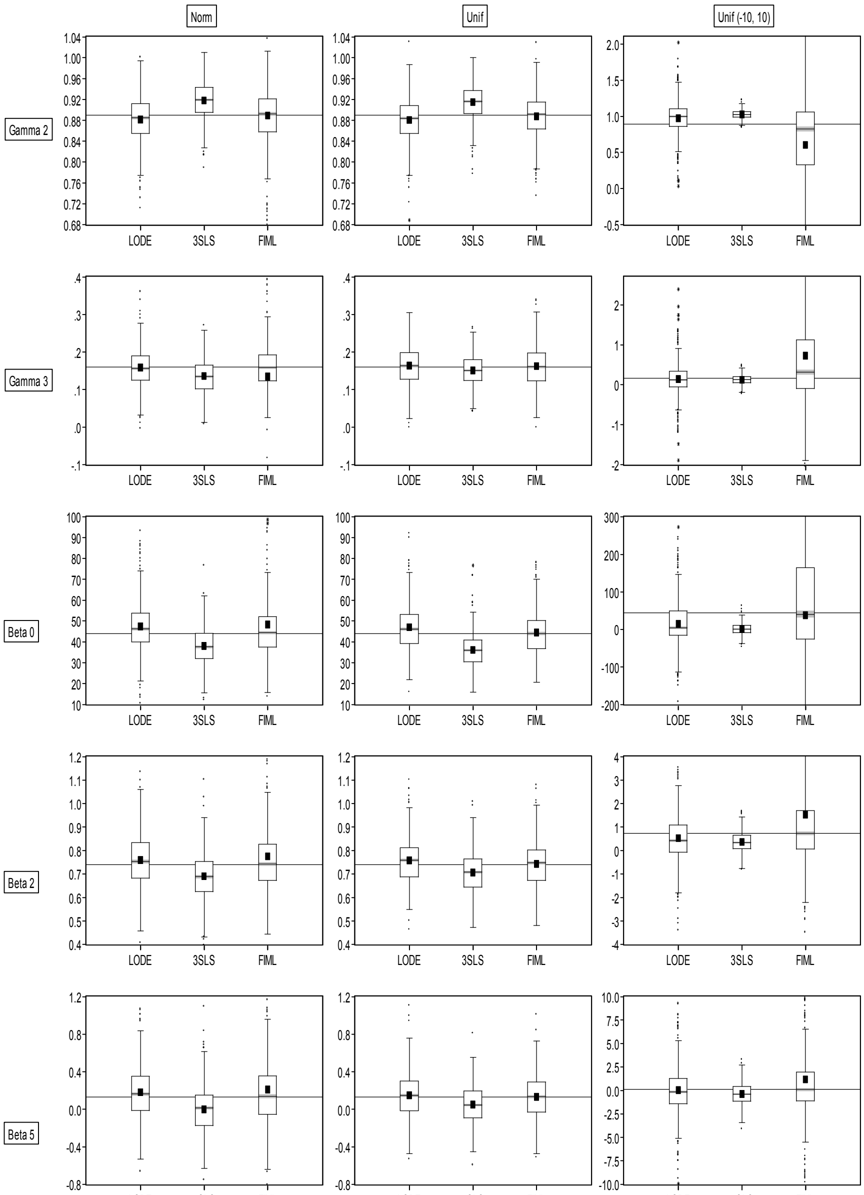


*Figure 1 - Equation 1*

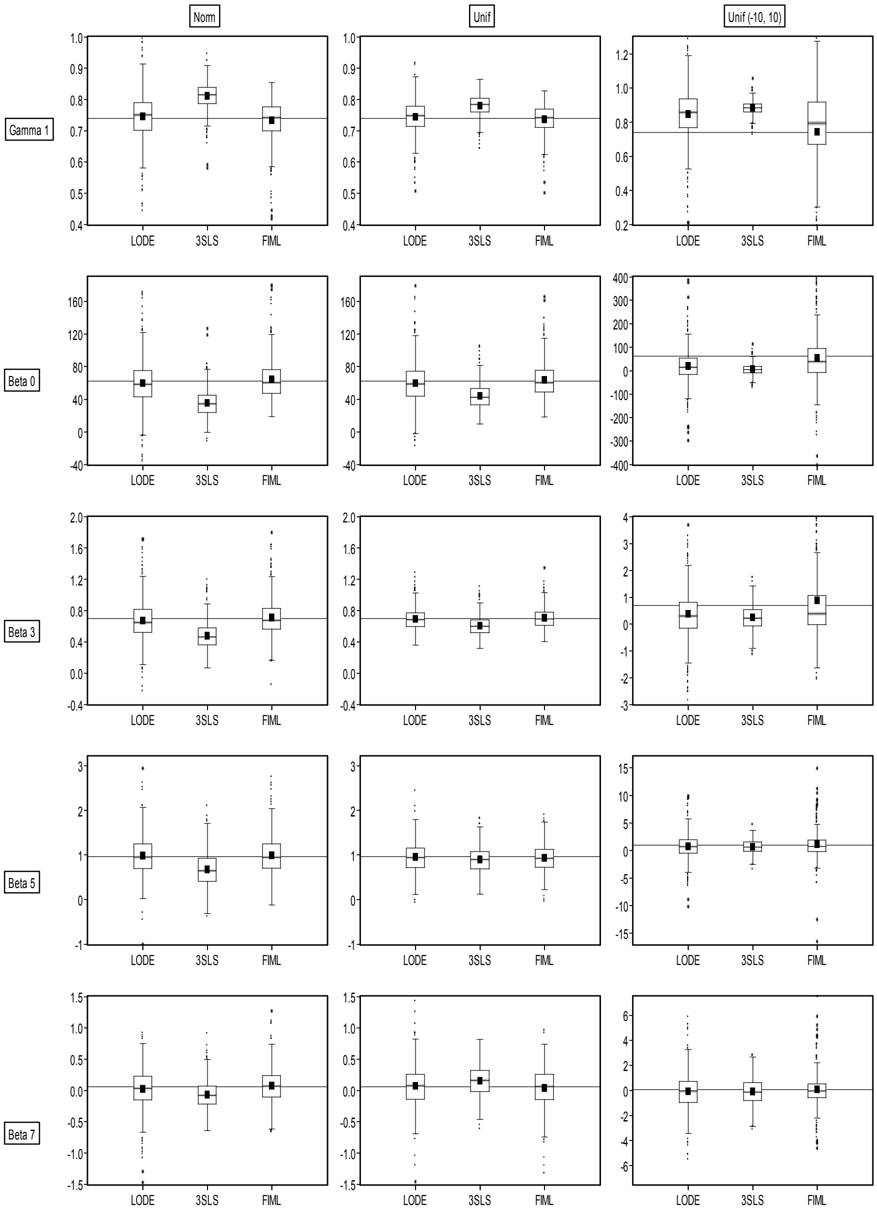


*Figure 2- Equation 2*

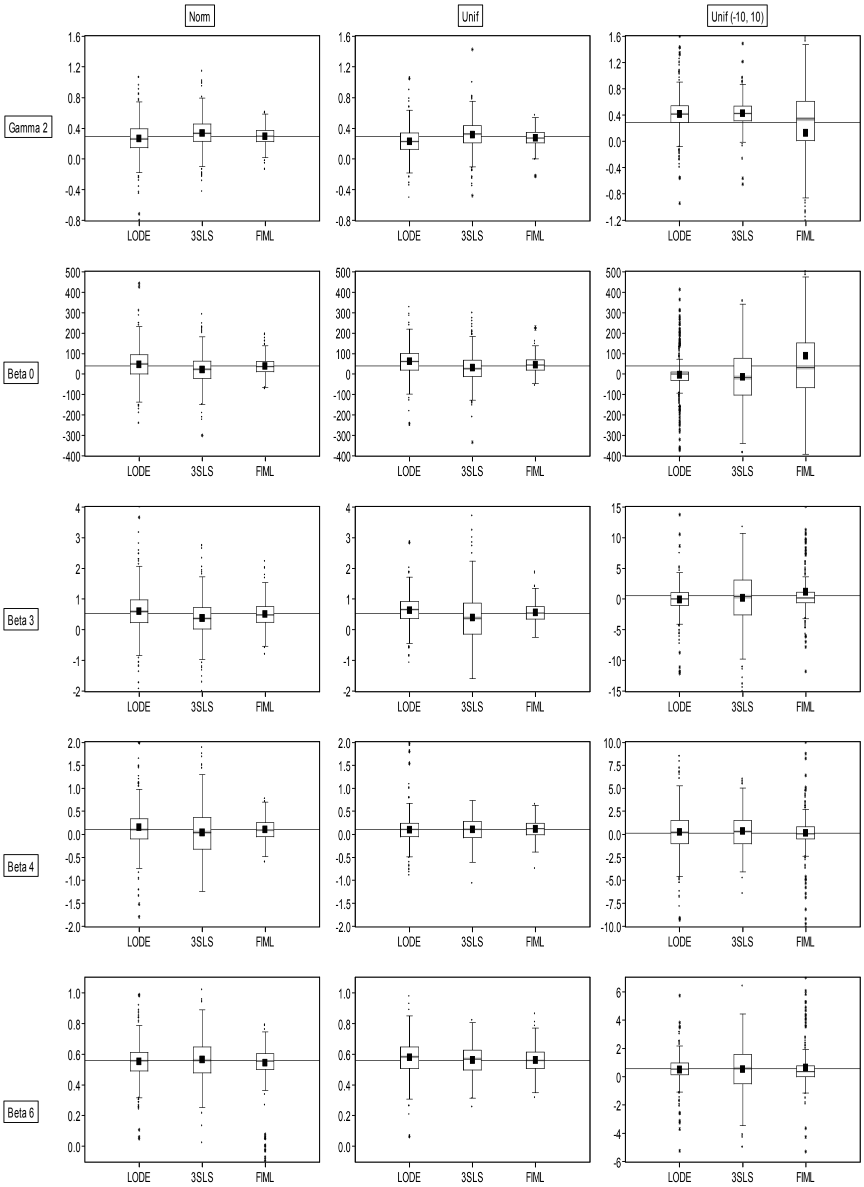


*Figure 3 - Equation 3*